

Fig. 3. Temperature profile in conjugate problem (gas-body): a) accurate solution; b) approximate solution. T, °K.

NOTATION

x, y, coordinates; u, v, velocity components; ρ , density; p, pressure; \bar{c}_T , element concentrations; c_i , mass concentrations of components; x_i , molar concentrations of components; \bar{K}_{τ} , diffusional mass flows of elements; K_i , diffusional mass flows of components; T, temperature; c_{peff} , total specific heat of mixture; M_{τ} , molecular weights of elements; M_i , molecular weights of components; M, molecular weight of mixture; \tilde{R} , universal gas constant; $D_{ij}(1)$, diffusion coefficient of binary mixture; D_i , generalized diffusion coefficients; μ , viscosity; λ , heat conduction.

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CONJUGATE HEAT-EXCHANGE PROBLEM IN THE FLOW OF A STREAM OF DISSOCIATED AIR OVER A BLUNT AXISYMMETRIC BODY

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An algorithm is constructed and the results of a numerical solution are presented for the conjugate problem of nonsteady heat exchange in the vicinity of the critical point of a blunt axisymmetric body during its interaction with a hypersonic airstream.

The nonsteady thermal interaction of an oncoming stream of liquid or gas with a solid body is characterized by the fact that the thermal boundary conditions at the surface over which the flow occurs vary with time. And these conditions are not known in advance but must be found in the course of the solution of the problem of nonsteady heat exchange.

The most general approach to the solution of problems of nonsteady convective heat exchange in a gas—solid body system consists in treating them as conjugate [1, 2]. A system of equations consisting of the equations for the nonsteady boundary layer for the gaseous zone and the heat-conduction equation for the solid

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body is analyzed in this case. Boundary conditions of the fourth kind, equality of the temperatures and heat fluxes for the gaseous and solid zones at the interface, are set up at the surface over which the flow occurs. Such a statement of the problem allows one to estimate the thermal interaction of the gas and the solid body.

The numerical solution of the conjugate problem of nonsteady heat exchange in the vicinity of the critical point of a blunt axisymmetric body over which a stream of dissociated air flows is discussed in the present report. The problem was solved under the following assumptions: 1) The boundary layer is assumed to be laminar; 2) the flow of a "frozen" stream of a gas mixture with a constant heat capacity C_p over a body is analyzed; 3) the high-temperature air consists of five components: N_2 , O_2 , NO, N, and O; 4) all the coefficients of binary diffusion are equal to each other; 5) the condition $\delta \ll R$ is valid for the envelope of the solid body (δ is the thickness and R is the blunting radius). The latter assumption allows one to use a one-dimensional heat-conduction equation [3].

The assumptions adopted allow a considerable simplification of the mathematical side of the problem and at the same time they make is possible to analyze sufficiently fully the properties of the nonsteady heat exchange in the gas—solid body system.

For the case under consideration the system of equations for the nonsteady gaseous boundary layer, without allowance for baro- or thermodiffusion, has the form [4]

$$\frac{\partial}{\partial t}(\rho r) + \frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial y}(\rho v r) = 0,$$
 (1)

$$\rho\left(\frac{\partial u}{\partial t} + u \ \frac{\partial u}{\partial x} + v \ \frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\mu \ \frac{\partial u}{\partial y}\right),\tag{2}$$

$$\frac{\partial P}{\partial y} = 0, \tag{3}$$

$$\frac{\partial C_k}{\partial t} + u \quad \frac{\partial C_k}{\partial x} + v \quad \frac{\partial C_k}{\partial y} - D_{12} \quad \frac{\partial C_k}{\partial y} = 0, \ k = 1, \ 2, \ 3, \ 4,$$
(4)

$$\rho C_P \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial P}{\partial t} , \qquad (5)$$

$$\sum_{k=1}^{5} C_k = 1,$$
 (6)

$$P = \frac{1}{M} \rho \tilde{R} T.$$
⁽⁷⁾

The boundary conditions are written as follows:

$$y = 0: \ u = v = 0, \ C_k = C_{k_W}(t), \ k = 1, \ 2, \ \dots, \ 5, \ T = T_W(t) \\ y \to \infty: \ u \to u_e(t), \ v \to 0, \ C_k \to C_{k_e}(t), \ k = 1, \ 2, \ \dots, \ 5, \ T \to T_e(t) \end{cases}$$
(8)

The process of heat transfer in the solid body is described by the heat-conduction equation

$$\rho_T C_T \frac{\partial T}{\partial t} = \frac{\partial}{\partial y} \left(\lambda_T \frac{\partial T}{\partial y} \right), \quad -\delta < x < 0, \quad t > 0,$$
(9)

with the following condition at the inner wall:

$$\lambda_T - \frac{\partial T(-\delta, t)}{\partial y} = q_{in}(t) \quad \text{or} \quad T(-\delta, t) = T_{in}(t).$$
(10)

The following conditions of the fourth kind are analyzed at the surface over which the flow occurs:

$$T_{W_{\Gamma}}(t) = T_{W_{T}}(t)$$

$$\left(\lambda \quad \frac{\partial T}{\partial y}\right)_{W} = \left(\lambda_{T} \quad \frac{\partial T}{\partial y}\right) - \varepsilon \sigma T_{W}^{4}(t)$$
(11)

In the vicinity of the critical point we introduce the dimensionless variables [5, 6]

$$\eta = \sqrt{\frac{2\beta(t)}{\mu_*\rho_*}} \int_0^s \rho dy, \quad \xi = \frac{1}{4} \mu_*\rho_*\beta(t) x^4$$

$$\frac{\partial f}{\partial \eta} = \frac{\mu(y, t)}{\mu_e(t)}, \quad \theta = \frac{T(y, t)}{T_e(t)}, \quad \tau = \int_0^t \beta(t) dt,$$
(12)

where $\mu *$ and $\rho *$ are the viscosity and density of the gas mixture at some fixed values of the temperature T_* and pressure P_* .

In Eqs. (12) it is assumed that in the vicinity of the critical point there is a linear law of velocity distribution at the limit of the boundary layer:

$$u_e(x, t) = \beta(t) x, \ \beta(t) = \left[\frac{\partial u_e(x, t)}{\partial x}\right]_{x=0}$$

In the numerical determination of the heat-exchange parameters it is necessary to calculate rather exactly the gradients of the temperature and of the concentrations of the components of the gas mixture at the surface being heated. In the present report this is achieved through the use of an integration step which is variable over the spatial coordinate. Such an approach permits a considerable increase in the accuracy of the calculations for the same volume of computations. We will use the following transformation of coordinates:

$$v = \frac{1}{s} \ln \left[1 + (\exp(s) - 1) \frac{\eta}{\eta_e} \right], \quad z = \frac{1}{r} \ln \left[1 + (\exp(r) - 1) \frac{y}{b} \right],$$
(13)

where s > 0 and r > 0 are the crowding parameters and $\eta_e(t)$ is the conventional thickness of the dynamic boundary layer. At each time the quantity $\eta_e(t)$ is determined from the condition

$$\frac{\partial w}{\partial v} \leqslant \varepsilon$$

of smooth conjugation of the dimensionless velocity profile with the inviscid stream [7], where $\varepsilon > 0$ is a small fixed quantity.

With a fixed number of steps across the dynamic boundary layer the transformation (13), through the choice of the parameters s and r, allows one to achieve the required bunching of the difference grid along the physical coordinate η in the region near the wall.

Using Eqs. (12) and (13), we can represent the system of equations (1)-(11) in the following way:

$$\frac{\partial f}{\partial v} = \frac{\operatorname{sn}_{e} \exp\left(\operatorname{sv}\right)}{\exp\left(\operatorname{s}\right) - 1} \quad w,$$

$$\frac{\partial w}{\partial \tau} = \frac{2 \left(\exp\left(\operatorname{s}\right) - 1\right)^{2}}{\operatorname{s}^{2} \eta_{e}^{2} \exp\left(\operatorname{sv}\right)} \cdot \frac{\partial}{\partial v} \left(\frac{l}{\exp\left(\operatorname{sv}\right)} \cdot \frac{\partial w}{\partial v}\right)$$

$$+ (a + w)w + \frac{\exp\left(\operatorname{sv}\right) - 1}{\operatorname{sn}_{e} \exp\left(\operatorname{sv}\right)} \left(2 \frac{\exp\left(\operatorname{s}\right) - 1}{\exp\left(\operatorname{sv}\right) - 1} \quad f - \frac{1}{2} \quad \operatorname{an}_{e} + \frac{d\eta_{e}}{d\tau}\right)$$

$$\times \frac{\partial w}{\partial v} + (1 + a) \quad \frac{M_{e}}{M} \quad \theta,$$

$$\frac{\partial C_{h}}{\partial \tau} = \frac{2 \left(\exp\left(\operatorname{s}\right) - 1\right)^{2}}{\operatorname{s}^{2} \eta_{e}^{2} \exp\left(\operatorname{sv}\right)} \cdot \frac{\partial}{\partial v} \left(\frac{l \operatorname{Le}_{12}}{\operatorname{Pr} \exp\left(\operatorname{sv}\right)} \cdot \frac{\partial C_{h}}{\partial v}\right)$$

$$+ w \quad \frac{\partial C_{k_{e}}}{\partial \tau} + \frac{\exp\left(\operatorname{sv}\right) - 1}{\operatorname{sn}_{e} \exp\left(\operatorname{sv}\right)}$$

$$\times \left(2 \quad \frac{\exp\left(\operatorname{s}\right) - 1}{\exp\left(\operatorname{sv}\right) - 1} \quad f - \frac{1}{2} \quad a\eta_{e} + \frac{d\eta_{e}}{d\tau}\right) \quad \frac{\partial C_{h}}{\partial v}, \quad k = 1, 2, \dots, 5,$$

$$\frac{\partial \theta}{\partial \tau} = \frac{2 \left(\exp\left(\operatorname{s}\right) - 1\right)^{2}}{\operatorname{s}^{2} \eta_{e}^{2} \exp\left(\operatorname{sv}\right)} \cdot \frac{\partial}{\partial v} \left(\frac{l}{\operatorname{Pr} \exp\left(\operatorname{sv}\right)} \cdot \frac{\partial \theta}{\partial v}\right)$$
(15)

$$+ \frac{\exp(sv) - 1}{s\eta_e \exp(sv)} \left(2 \frac{\exp(s) - 1}{\exp(sv) - 1} f - \frac{1}{2} a\eta_e + \frac{d\eta_e}{d\tau} \right) \frac{\partial \theta}{\partial v} \\ + \left[\frac{\tilde{R}}{MC_P P_e} \cdot \frac{\partial P_e}{\partial \tau} \left(\frac{M_e}{M} - w \right) - \frac{1}{T_e} \frac{\partial T_e}{\partial \tau} (1 - w) \right] \theta,$$
(16)

$$\rho_{T}C_{T} \frac{\partial \theta}{\partial \tau} = \frac{\exp\left(r\right) - 1}{r^{2}\delta^{2}\beta \exp\left(rz\right)} \cdot \frac{\partial}{\partial z} \left(\frac{\lambda_{T}}{\exp\left(rz\right)} \cdot \frac{\partial \theta}{\partial z}\right) - \rho_{T}C_{T} \frac{\partial T_{e}}{\partial \tau} \theta, \qquad (17)$$
$$-1 < z < 0, \quad \tau > 0,$$

$$-T_e \lambda_T - \frac{\exp(r) - 1}{r\delta} \left. \frac{\partial \theta}{\partial z} \right|_{z=-1} = q_{in}(\tau) \text{ or } \theta(-1, \tau) = \frac{T_{in}(\tau)}{T_e(\tau)}, \qquad (18)$$

$$\mathbf{v} = 0: f = w = 0, \ C_{k} = C_{k_{W}}(\mathbf{\tau}), \ k = 1, \ 2, \ \dots, \ 5, \ \theta = \frac{T_{W}(\mathbf{\tau})}{T_{e}(\mathbf{\tau})} \bigg\},$$
(19)

$$v = 1: w = 1, C_{k} = C_{ke}(\tau), k = 1, 2, \dots, 5, \theta = 1$$

$$\frac{\lambda_{TW} \operatorname{Pr}_{W}(\exp(r) - 1)}{C_{P} l_{W} r \delta \exp(r) \sqrt{2\beta(\tau)} \mu_{*} \rho_{*}} \left(\frac{\partial \theta}{\partial z}\right)_{W} = \frac{\exp(s) - 1}{s \eta_{e}} \left(\frac{\partial \theta}{\partial v}\right)_{W}$$

$$+ \frac{(\operatorname{Le}_{12})_{W}(\exp(s) - 1)}{s \eta_{e} C_{P} T_{e}} \sum_{k=1}^{5} I_{k} \left(\frac{\partial C_{k}}{\partial v}\right) - \frac{\operatorname{Pr}_{W} \varepsilon \sigma T_{e}^{3}}{l_{W} C_{P} \sqrt{2\beta(\tau)} \mu_{*} \rho_{*}} \theta_{W}^{4}, \qquad (20)$$

where $l = \mu \rho / \mu_* \rho_*$, $a = (1/\beta^2) (d\beta/dt)$ is a coefficient allowing for the time variation of the parameters of the external stream (u_e, P_e, T_e), Pr = $\mu C_P / \lambda$ is the Prandtl number, and Le₁₂ = $\rho D_{12} C_P / \lambda$ is the Lewis number.

Formally, Eqs. (14)-(16) have the same form

$$\frac{\partial \vartheta}{\partial \tau} = b \; \frac{\partial}{\partial \nu} \left(d \; \frac{\partial \vartheta}{\partial \nu} \right) + g \; \frac{\partial \vartheta}{\partial \nu} + \varphi \vartheta + \Psi, \tag{21}$$

where the coefficients depend on the functions sought. In operator form (21) can be represented as

$$\frac{\partial \vartheta}{\partial \tau} = L\vartheta + \Psi, \ L = b \ \frac{\partial}{\partial \nu} \left(d \ \frac{\partial}{\partial \nu} \right) + g \ \frac{\partial}{\partial \nu} + \varphi.$$
(22)

We introduce the difference grid

$$\{z_i=ih_1, i=0, 1, \ldots, m, v_i=ih, i=0, 1, \ldots; \tau_j=\sum_{k=1}^{j}\Delta\tau_k, j=0, 1, \ldots\},\$$

where $h_1 = 1/m$, h = 1/n, m and n are the numbers of partition intervals in the solid body and across the dynamic boundary layer, respectively, while the step $\Delta \tau_j$ in dimensionless time is calculated from the equation

$$\Delta \tau_j = \frac{\beta_j + \beta_{j-1}}{2} \ \Delta t.$$

To represent Eq. (21) in finite-difference form we use the implicit monotonic scheme of approximation [8]. In this case instead of the operator L in (22) we consider the operator

$$\tilde{L} = \varkappa b \; \frac{\partial}{\partial v} \left(d \; \frac{\partial}{\partial v} \right) \div g \; \frac{\partial}{\partial v} \; \div \varphi,$$

where $\kappa = 1/(1 + R)$ and R = h|g|/2d is the difference Reynolds number.

To approximate the convective term in Eq. (21) we use the relation

$$\begin{pmatrix} g \ \frac{\partial \Phi}{\partial v} \end{pmatrix}_{i} = \frac{g_{i} + |g_{i}|}{2d_{i}} \cdot \frac{d_{i} + d_{i+1}}{2} \cdot \frac{\Phi_{i+1} - \Phi_{i}}{h} + \frac{g_{i} - |g_{i}|}{2d_{i}} \cdot \frac{d_{i} + d_{i-1}}{2} \cdot \frac{\Phi_{i} - \Phi_{i-1}}{h} .$$

$$(23)$$

The advantage of such an approach is that "ripples" in the solution along the spatial coordinate do not arise for any values of the coefficient g. Despite the use of one-sided differences in (23), the entire scheme formally has the order of approximation $o(h^2)$ [8].

As a result of the transition to finite differences, Eqs. (14)-(16) are reduced, at each step in time, to systems of nonlinear algebraic equations with three-diagonal matrices, which have the form

$$A_{i}^{j} \Theta_{i-1}^{j} + B_{i}^{j} \Theta_{i}^{j} + D_{i}^{j} \Theta_{i+1}^{j} = F_{i}^{j}, \ i = 1, 2, \dots$$
(24)

The iterational process of solving Eqs. (24) will be set up using the method of asymptotic establishment [9]. We represent (24) in the matrix form

$$N^{j}\boldsymbol{\vartheta}^{j} = \boldsymbol{\vartheta}^{j}, \qquad (25)$$

where N^j is a three-diagonal matrix.

The solution of Eq. (25) with fixed values of the unknown function at the boundary nodes is determined as the asymptotic solution of the nonsteady process

$$\frac{\partial \vartheta^{j}}{\partial \sigma} = N^{j} \vartheta^{j} - \theta^{j}, \qquad (26)$$

where σ is the "fictitious" time.

Equation (26) can be represented in the difference form, using the implicit scheme of approximation:

$$\frac{\boldsymbol{\vartheta}_{i}^{j(p)}-\boldsymbol{\vartheta}^{j(p-1)}}{\Delta\sigma}=A_{i}^{j(p-1)}\boldsymbol{\vartheta}_{i}^{j(p)}+B_{i}^{j(p-1)}\boldsymbol{\vartheta}_{i}^{j(p)}+D_{i}^{j(p-1)}\boldsymbol{\vartheta}_{i}^{j(p)}-F^{j(p-1)}$$

 \mathbf{or}

$$A_{i}^{j(p-1)}\vartheta_{i}^{j(p)} + \left(B_{i}^{j(p-1)} - \frac{1}{\Delta\sigma}\right)\vartheta_{i}^{j(p)} + D_{i}^{j(p-1)}\vartheta_{i}^{j(p)} = F_{i}^{j(p-1)} - \frac{1}{\Delta\sigma}\vartheta_{i}^{j(p-1)},$$
(27)

where p is the number of the iteration and $\Delta \sigma$ is the step in "fictitious" time.

An implicit divergent scheme of approximation [8] is used for the heat-conduction equation (17). As a result, we obtain a system of nonlinear algebraic equations of the type of (24).

The difference analog of the equation of thermal balance (20) is represented in the following way:

$$\frac{h \operatorname{sy}_{e}^{l} \lambda_{TW}^{i} \operatorname{Pr}_{W}^{l} (\exp(r) - 1)}{C_{P} l_{W}^{i} r \delta \exp(r) (\exp(s) - 1) h_{1} \sqrt{2\beta^{j} \mu_{*} \rho_{*}}} \left(\theta_{-2}^{l} - 4\theta_{-1}^{l} + 3\theta_{W}^{i} \right)$$

$$= -\theta_{2}^{l} + 4\theta_{1}^{l} - 3\theta_{W}^{i} + \frac{(\operatorname{Le}_{12})_{W}^{i}}{C_{P} T_{e}^{l}} \sum_{k=1}^{5} I_{k}^{i} \left(-C_{k_{*}}^{l} + 4C_{k_{*}}^{l} - 3C_{k_{W}}^{l} \right)$$

$$- \frac{\operatorname{Pr}_{W}^{i} \operatorname{eo}\left(T_{e}^{l}\right)^{3} 2h \operatorname{sy}_{e}^{l}}{l_{W}^{i} C_{P} (\exp(s) - 1) \sqrt{2\beta^{j} \mu^{*} \rho^{*}}} \left(\theta_{W}^{i} \right)^{4}.$$
(28)

Using the difference approximation of the energy equation (16) and the heat-conduction equation (17), to eliminate the quantities $\theta \dot{J}_{-2}$ and $\theta \dot{J}_{2}$ from (28) this equation is reduced to the form

$$A'_{lv}\theta'_{-1} + B'_{lv}\theta'_{lv} + D'_{lv}\theta'_{1} = F'_{lv} .$$
⁽²⁹⁾

The system of algebraic equations (27) and (24) are solved using the trial-run method. In doing this the heat-conduction equation and the energy equation are treated as one equation and the trial run is carried straight through, starting from the inner surface of the solid body to the outer limit of the boundary layer.

The boundary conditions for the diffusion equations were determined on the assumption that an equilibrium composition of the air-gas mixture is realized at the external limit of the boundary layer on the surface over which the flow occurs. The physical properties of the dissociated air were calculated with allowance for their dependence on the temperature and pressure. Approximate functions, approximating the results of [10], were used for the calculation.



Fig. 1. Variation of convective heat-flux density during nonsteady heating of bodies of the same thickness ($\delta = 25 \text{ mm}$): 1) fused quartz, $\lambda = 6.12 \cdot 10^{-3} \text{ kW/m} \cdot ^{\circ}\text{K}, \rho \text{ C} = 3094 \text{ kJ/m}^3$; 2) graphite, $\lambda = 0.047 \text{ kW/m} \cdot ^{\circ}\text{K}, \rho \text{ C} = 3225 \text{ kJ/m}^3$; 3) copper, $\lambda = 0.392 \text{ kW/m} \cdot ^{\circ}\text{K}, \rho \text{ C} = 3368 \text{ kJ/m}^3$; 4) baked quartz, $\lambda = 7.21 \cdot 10^{-4} \text{ kW/m} \cdot ^{\circ}\text{K}, \rho \text{ C} = 2060 \text{ kJ/m}^3$. q_W , kW/m²; $T_W, ^{\circ}\text{K}$.

The steady-state solution of the problem described above at the time $\tau_0 = 0$ was used as the initial condition.

The iteration process is ended when a given level of accuracy is reached. Since the initial quantities of the problem are thermal parameters, it is desirable to construct the criterion for ending the iterations on their basis. In the present work this condition is the inequality

$$|q_{W}^{(p)} - q_{W}^{(p-1)}| \leqslant \varepsilon_{1} |q_{W}^{(p)}|$$
(30)

for the heat fluxes, where q_{W} is the convective heat flux and $\epsilon_{1} > 0$ is a given small quantity.

The value of the function $\eta_{e}(\tau)$ at the j-th time is refined in each iteration using the difference analog of Eq. (14) with the condition of the equality

$$\frac{\partial w}{\partial v} = \varepsilon. \tag{31}$$

The derivative $\partial w / \partial v$ is calculated at the nodes of the difference grid

$$\left(\frac{\partial w}{\partial v}\right)_n = \frac{w_{n-2} - 4w_{n-1} + 3w_n}{2h}, \quad \left(\frac{\partial w}{\partial v}\right)_i = \frac{w_{i+1} - w_{i-1}}{2h}, \quad i < n,$$

and is approximated by a piecewise linear function. If $(\partial w/\partial \nu)_n > \varepsilon$, then η_e^j is increased by an amount

$$\Delta = \Delta \eta_n \left[\frac{\left(\frac{\partial w}{\partial v} \right)_n}{\varepsilon} - 1 \right], \text{ where } \Delta \eta_n = \eta_n - \eta_{n-1}.$$

In the zeroth approximation we set $\eta_e j = \eta_e j^{-1}$.

The algorithm described for the numerical solution of the conjugate problem of heat exchange was realized in the form of an ALGOL program for a BÉSM-6 computer, on which a number of calculations were made. In particular, the effect of the time rate of change $dT_W/d\tau$ of the temperature of the surface being heated on the convective heat flux delivered to the body was studied numerically. The heating of models having the same thickness $\delta = 25$ mm but made of different materials was analyzed. The parameters of the oncoming stream remained constant in time and were $P_e = 2 \text{ kg/cm}^2$, $T_e = 6000^{\circ}\text{K}$, $u_e = 3000 \text{ m/sec}$, and $T_0 = 273^{\circ}\text{K}$.

The results of the calculations are presented in Fig. 1. Different functions of the temperature of the surface being heated, and consequently different time functions of the convective heat flux, are realized in the process of heating for the different models (Fig. 1a). However, the function $q_W(T_W)$, which is common to the different materials, has a rectilinear character (Fig. 1b). This fact indicates that the heat-transfer coefficient in the vicinity of the critical point of a blunt axisymmetric body over which a hypersonic airstream flows is practically independent of the value of the derivative $dT_W/d\tau$.

We note that analogous results also occurred for other model thicknesses and values of the parameters of the oncoming stream. Here the maximum value of the derivative $dT_w/d\tau$ reached in the numerical experiments was $10^5 \, {}^{\circ}\text{K}/\text{sec.}$

NOTATION

t, τ , time; x, y, coordinates; η , ξ , dimensionless coordinates; u, v, components of the velocity vector; ρ , density; P, pressure; C_k, concentration by weight of k-th component; D₁₂, coefficient of binary diffusion; T, temperature; λ , coefficient of thermal conductivity; M, molecular weight; q, heat flux; ε , emissivity; h, h₁, integration step along the spatial coordinate; Δt , $\Delta \tau$, integration step in time; δ , thickness of solid body; μ , viscosity; I, total enthalpy; R_B, blunting radius of solid body; s, r, bunching parameters of difference grid; e, index of external limit of boundary layer; W, index of surface over which flow occurs.

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STEFAN-TYPE PROBLEM FOR SUBLIMATION

IN A POROUS BODY

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The mathematical formulation of the problem of heat and mass transfer during evaporation from a semiinfinite porous body consisting of parallel capillaries is given. The asymptotic solution of the problem is obtained for large and small time periods.

It was shown earlier [1] that the velocity of passage of the evaporation front v from capillaries (in the case of free-molecular regime of vapor flow) depends substantially on the depth of the evaporation zone in the porous body:

$$v = \frac{d\xi}{dt} = \frac{v_0}{1 + \xi/2r} , \qquad (1)$$

where [2]

$$v_0 \approx a \exp\left\{-LA/RT\right\}.$$
 (2)

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